Matemática I

Cálculo Diferencial

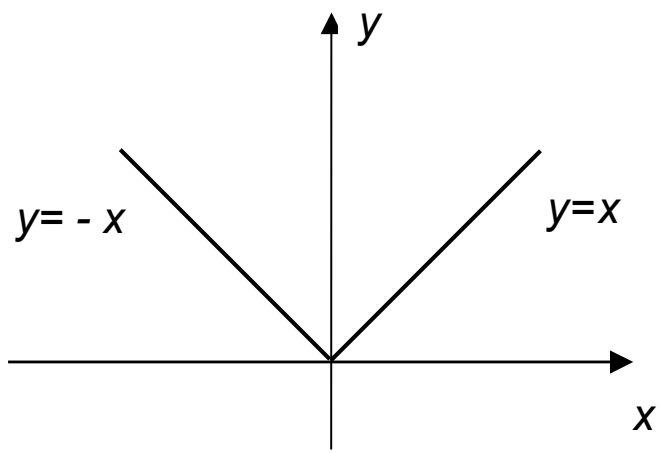
Engenharia de Telecomunicações e Informática

2021/2022

Teresa Araújo
Departamento de Matemática

Função Módulo

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

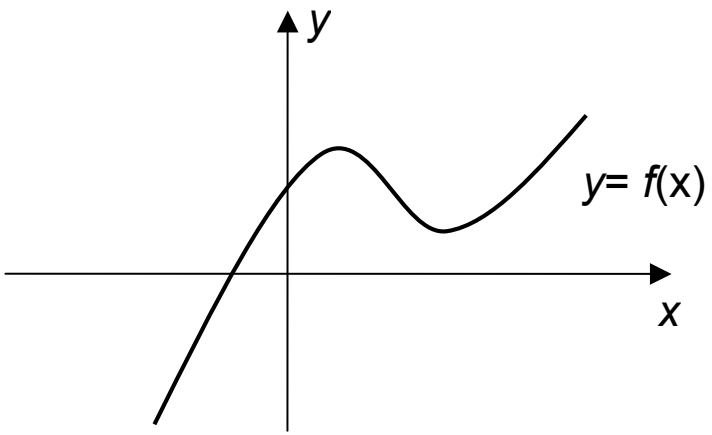


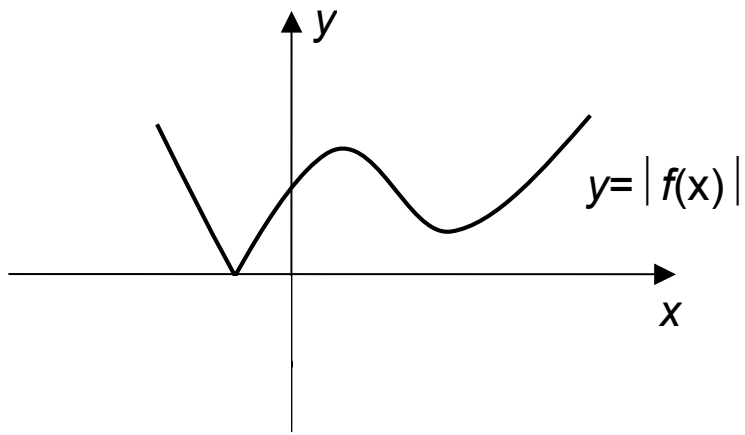
Domínio $D = \mathbb{R}$
Contradomínio $D' = \mathbb{R}_0^+$

Função Módulo

De uma forma geral

$$|f(x)| = \begin{cases} f(x), & f(x) \geq 0 \\ -f(x), & f(x) < 0 \end{cases}$$





Função Exponencial

$$y = a^x \qquad a \in \mathbb{R}^+ \setminus \{1\}$$

Domínio $D = \mathbb{R}$
Contradomínio $D' = \mathbb{R}^+$

Propriedades

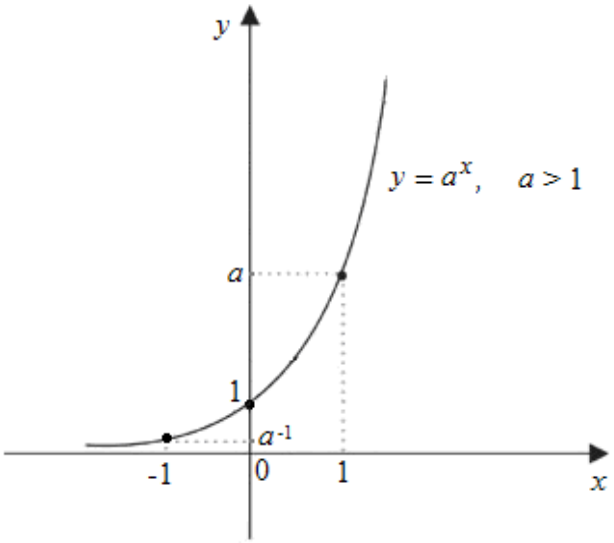
$$a^u \cdot b^u = (a \cdot b)^u$$
$$a^u \cdot a^v = a^{u+v}$$
$$\frac{a^u}{a^v} = a^{u-v}$$
$$(a^u)^v = a^{u \cdot v}$$

$$\sqrt[u]{a} = a^{1/u} \qquad\qquad a^{-u} = \frac{1}{a^u}$$

Função Exponencial

$$y = a^x \qquad a \in \mathbb{R}^+ \setminus \{1\}$$

$a > 1$

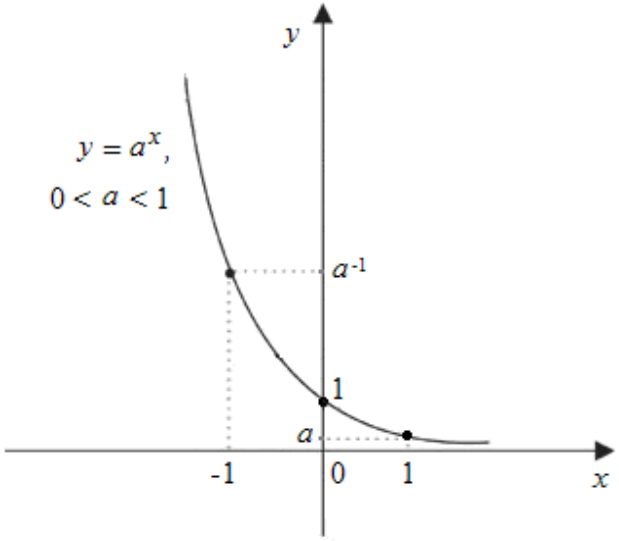


função crescente

$$\lim_{x \to -\infty} a^x = 0$$

$$\lim_{x \to +\infty} a^x = +\infty$$

$0 < a < 1$



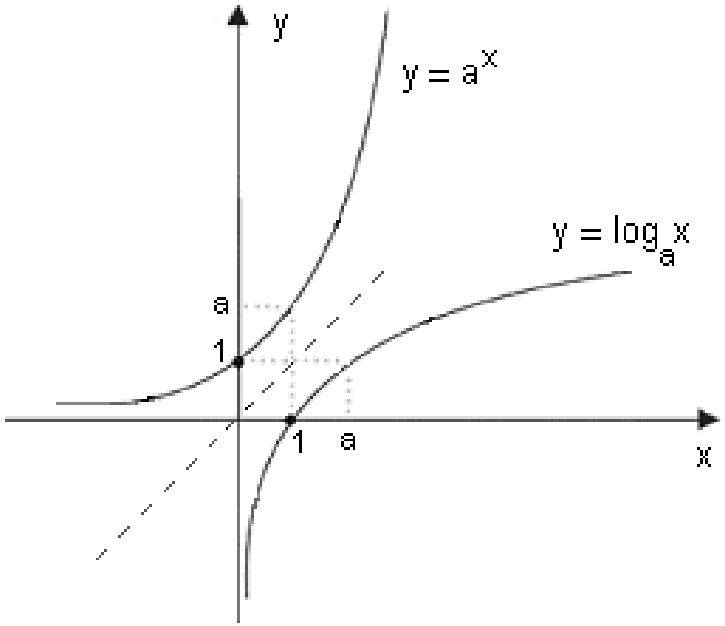
função decrescente

$$\lim_{x \to -\infty} a^x = +\infty$$

$$\lim_{x \to +\infty} a^x = 0$$

Funções Logarítmica e Exponencial

$$y = a^x \quad \Leftrightarrow \quad x = \log_a(y)$$



$$a^{\log_a(x)} = x$$

$$\log_a(a^x) = x$$

Caso particular

Número de Neper $e = 2,7182...$

Designa-se por **logaritmo neperiano** e escreve-se

$$y = \ln(x)$$

$$y = \ln(x) \quad \Leftrightarrow \quad x = e^y$$

Função Logarítmica

$$y = \log_a(x) \qquad a \in \mathbb{R}^+ \setminus \{1\}$$

Domínio $D = \mathbb{R}^+$
Contradomínio $D' = \mathbb{R}$

Propriedades

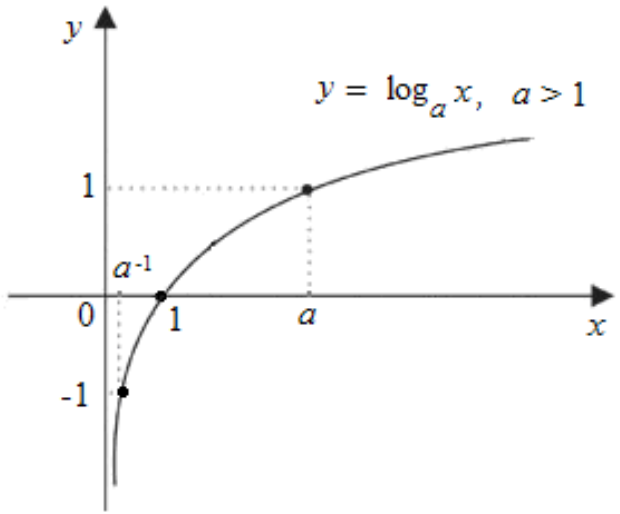
$$\log_a(u \cdot v) = \log_a(u) + \log_a(v)$$
$$\log_a\left(\frac{u}{v}\right) = \log_a(u) - \log_a(v)$$
$$\log_a(u^v) = v \log_a(u)$$

$$\log_a(a) = 1 \qquad \log_a(1) = 0 \qquad \log_a(b) = \frac{\log_c(b)}{\log_c(a)}$$

Função Logarítmica

$$y = \log_a(x)$$

$a > 1$

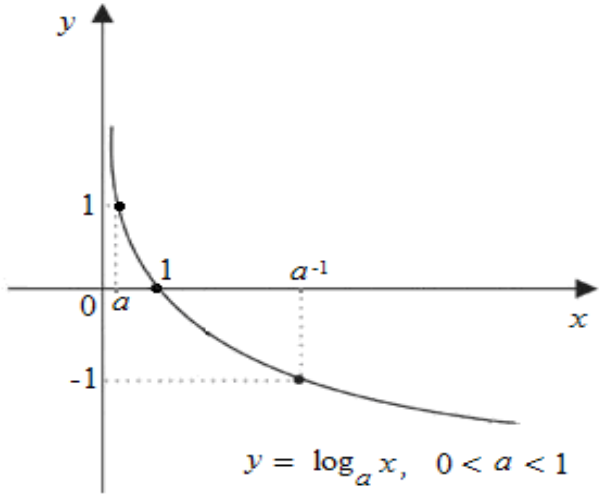


função crescente

$$\lim_{x \to 0^+} \log_a(x) = -\infty$$

$$\lim_{x \to +\infty} \log_a(x) = +\infty$$

$0 < a < 1$

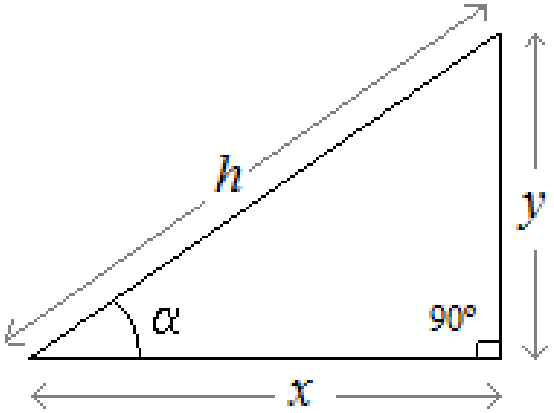


função decrescente

$$\lim_{x \to 0^+} \log_a(x) = +\infty$$

$$\lim_{x \to +\infty} \log_a(x) = -\infty$$

Funções Trigonométricas



$$\text{sen}(\theta) = \frac{y}{h} \qquad \text{cosec}(\theta) = \frac{h}{y}$$

$$\cos(\theta) = \frac{x}{h} \qquad \sec(\theta) = \frac{h}{x}$$

$$\text{tg}(\theta) = \frac{y}{x} \qquad \text{cotg}(\theta) = \frac{x}{y}$$

$$\text{tg}(\theta) = \frac{\text{sen}(\theta)}{\cos(\theta)} \qquad \text{cotg}(\theta) = \frac{\cos(\theta)}{\text{sen}(\theta)}$$

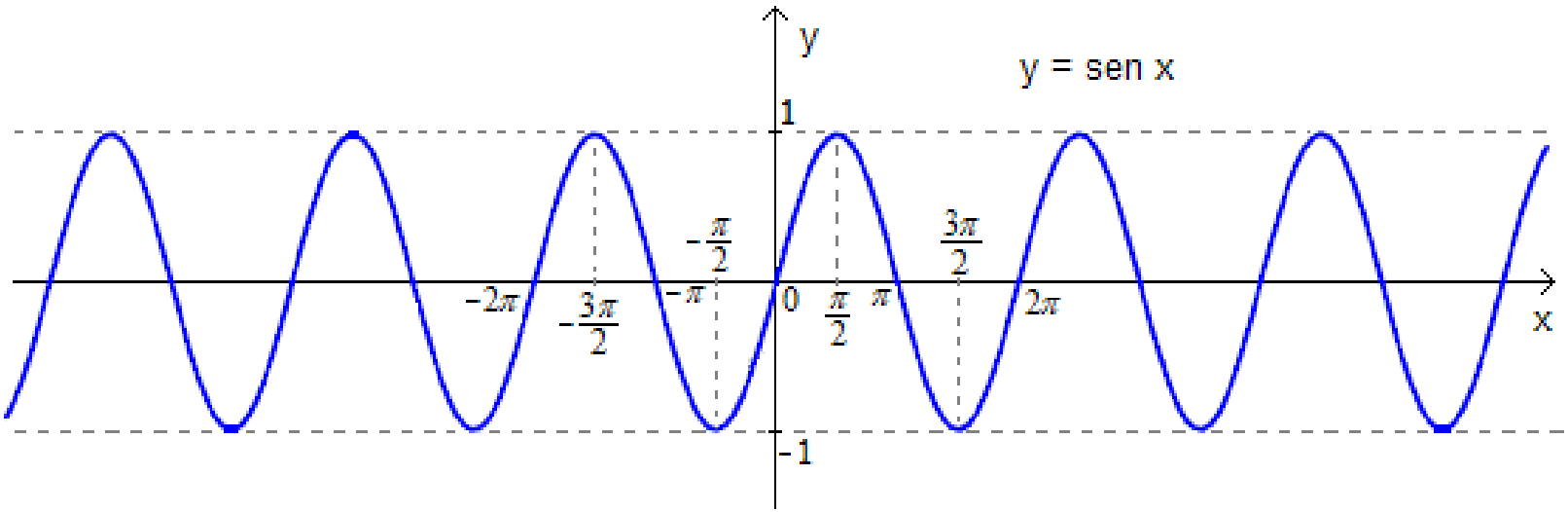
$$\sec(\theta) = \frac{1}{\cos(\theta)} \qquad \text{cosec}(\theta) = \frac{1}{\text{sen}(\theta)}$$

$$\text{sen}^2(\theta)+\cos^2(\theta) = 1 \qquad \text{tg}^2(\theta)+1 = \sec^2(\theta) \qquad 1+\text{cotg}^2(\theta) = \text{cosec}^2(\theta)$$

Tabela

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\text{sen}(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\text{tg}(\theta)$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞
$\text{cotg}(\theta)$	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0
$\sec(\theta)$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	∞
$\text{cosec}(\theta)$	∞	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1

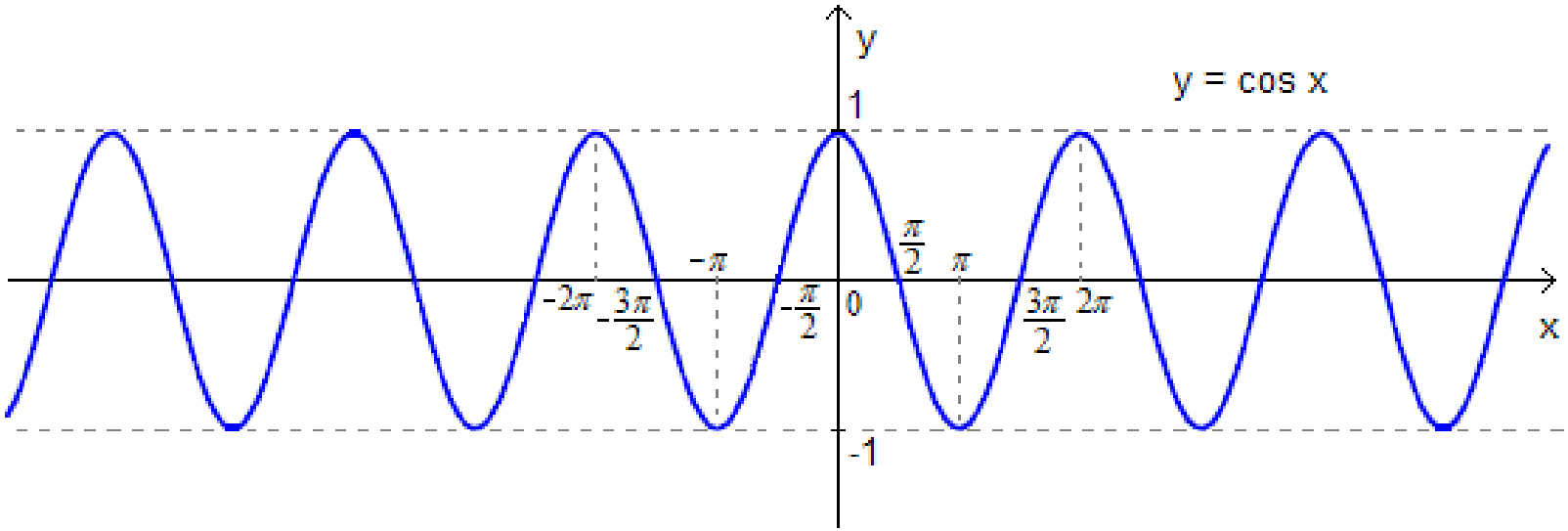
Função seno $y = \text{sen}(x)$



domínio $D = \mathbb{R}$
contradomínio $D' = [-1, 1]$

período 2π
função não injectiva
função ímpar
$\text{sen}(-x) = -\text{sen}(x)$

Função cosseno $y = \cos(x)$



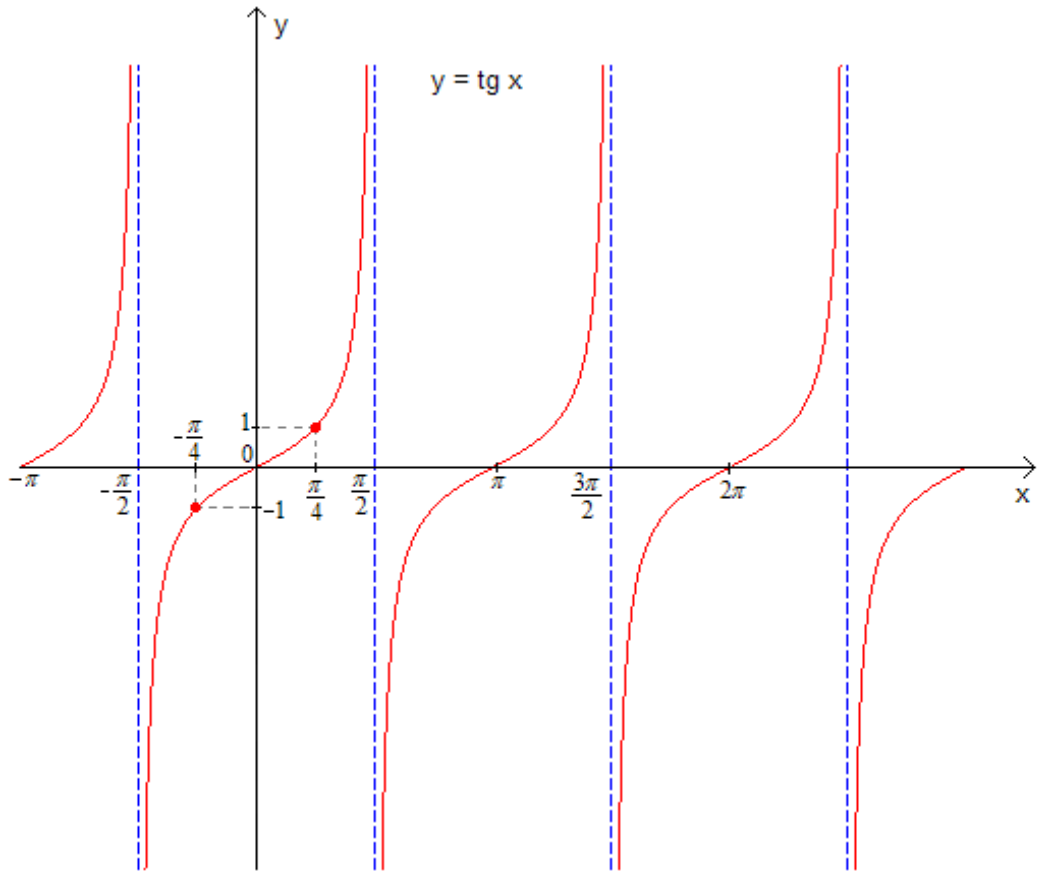
domínio $D = \mathbb{R}$
contradomínio $D' = [-1, 1]$

período 2π
função não injectiva
função par $\cos(-x) = \cos(x)$

Funções Trigonométricas

Função tangente

$$y = \text{tg}(x)$$



domínio $D = \left\{x \in \mathbb{R} : x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$

contradomínio $D' = \mathbb{R}$

assímptotas verticais $x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

período π

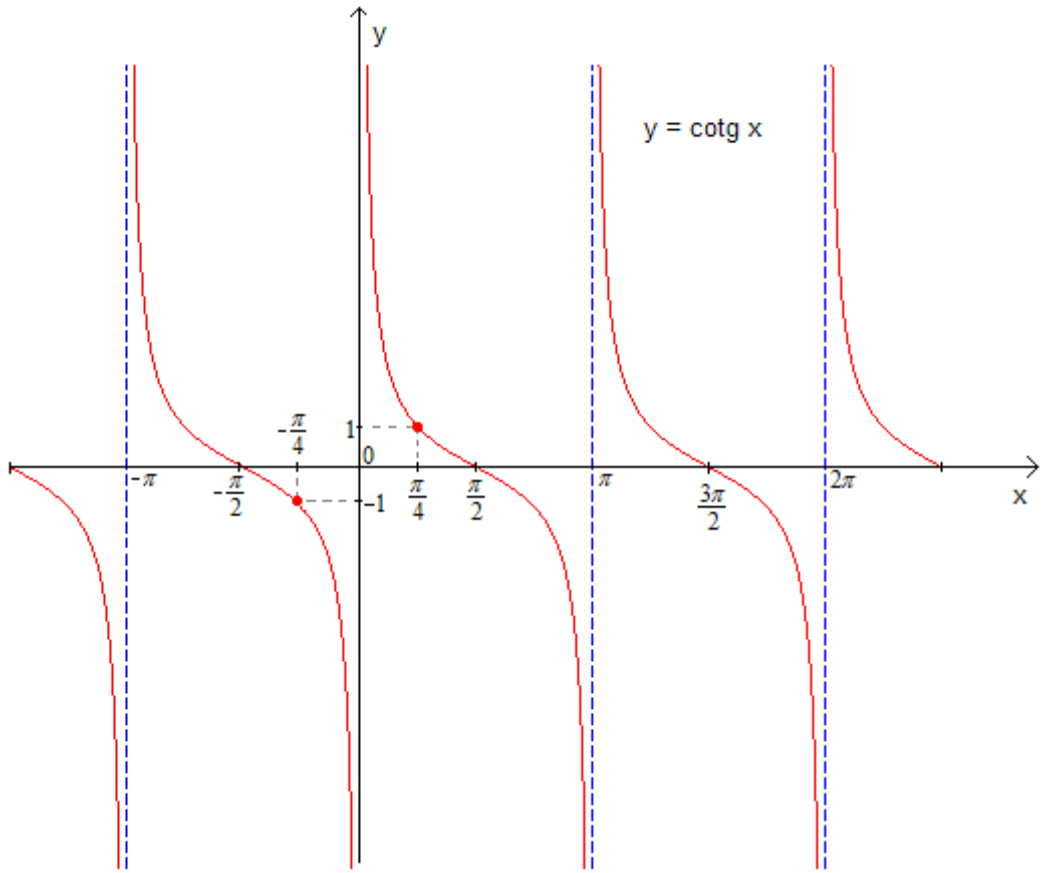
função não injectiva

função ímpar $\text{tg}(-x) = -\text{tg}(x)$

Funções Trigonométricas

Função cotangente

$$y = \text{cotg}(x)$$

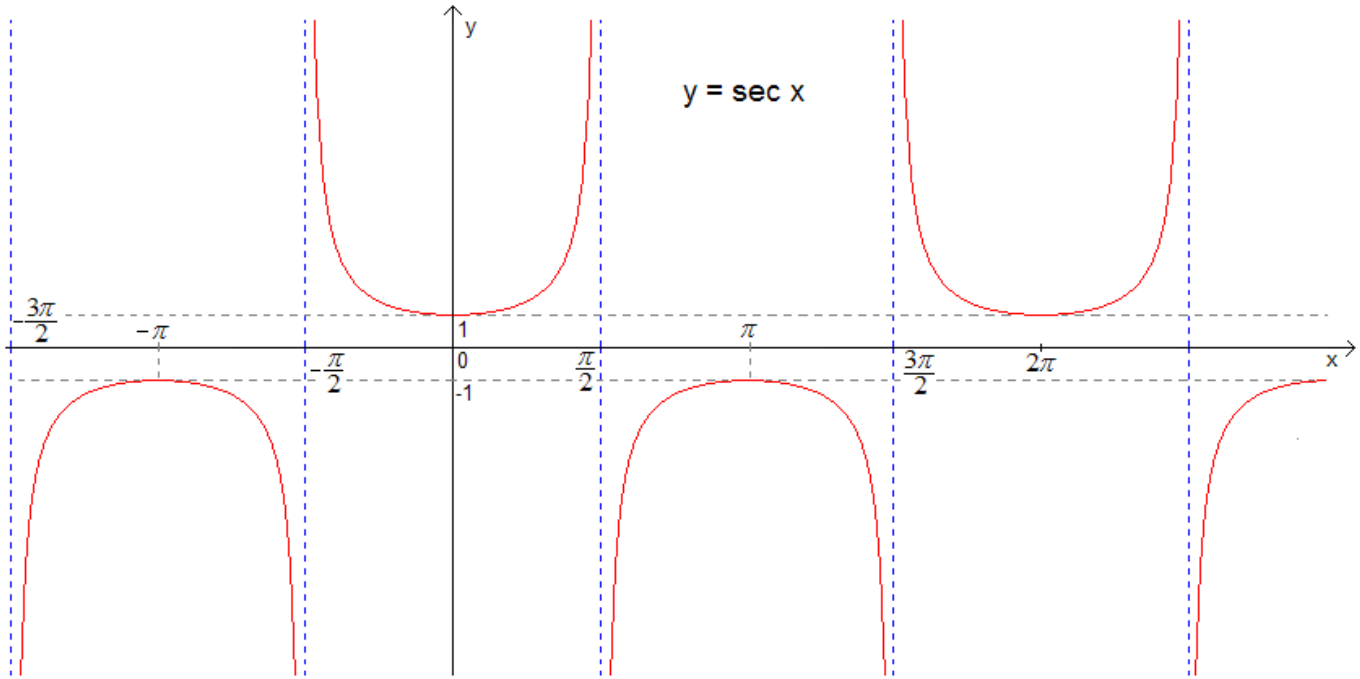


domínio $D = \{x \in \mathbb{R} : x \neq k\pi, k \in \mathbb{Z}\}$
contradomínio $D' = \mathbb{R}$
assímptotas verticais $x = k\pi, k \in \mathbb{Z}$

período π
função não injectiva
função ímpar
$\text{cotg}(-x) = -\text{cotg}(x)$

Funções Trigonométricas

Função secante $y = \sec(x)$



domínio $D = \left\{x \in \mathbb{R} : x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$

contradomínio $D' =]-\infty, -1] \cup [1, +\infty[$

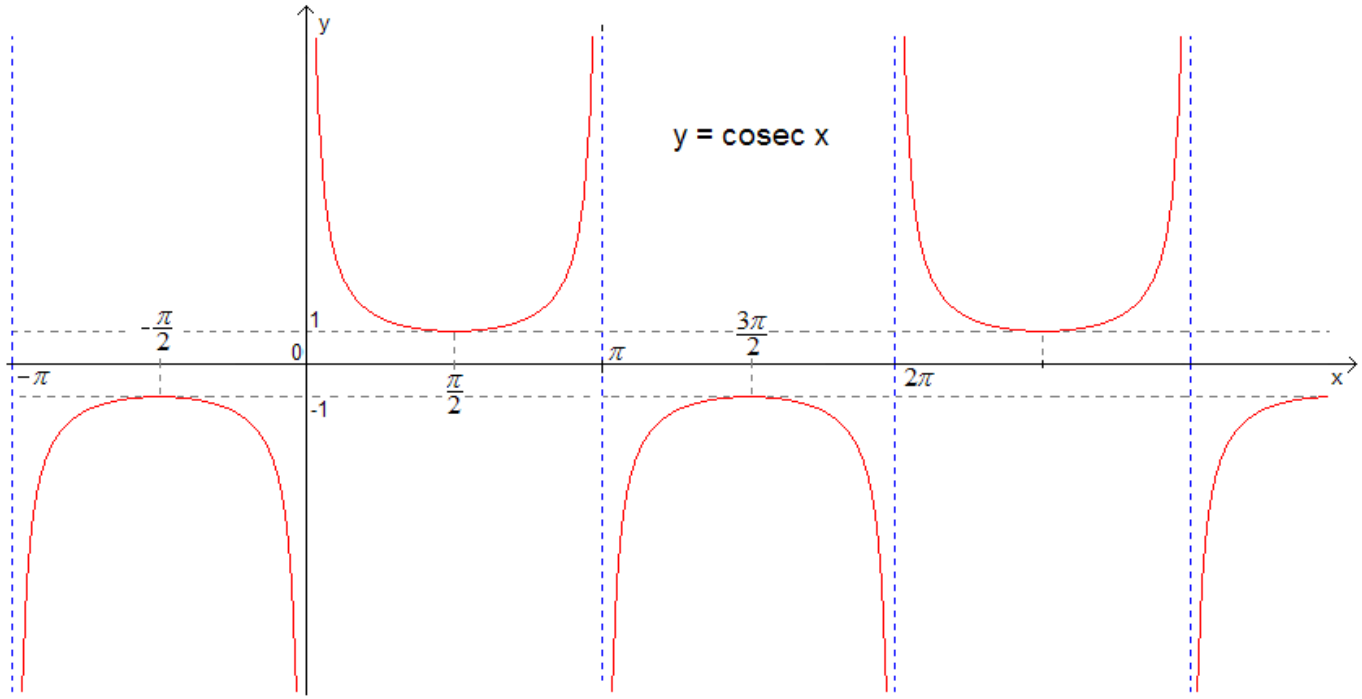
assímptotas verticais $x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

período 2π

função não injectiva

função par $\sec(-x) = \sec(x)$

Função cossecante $y = \text{cosec}(x)$

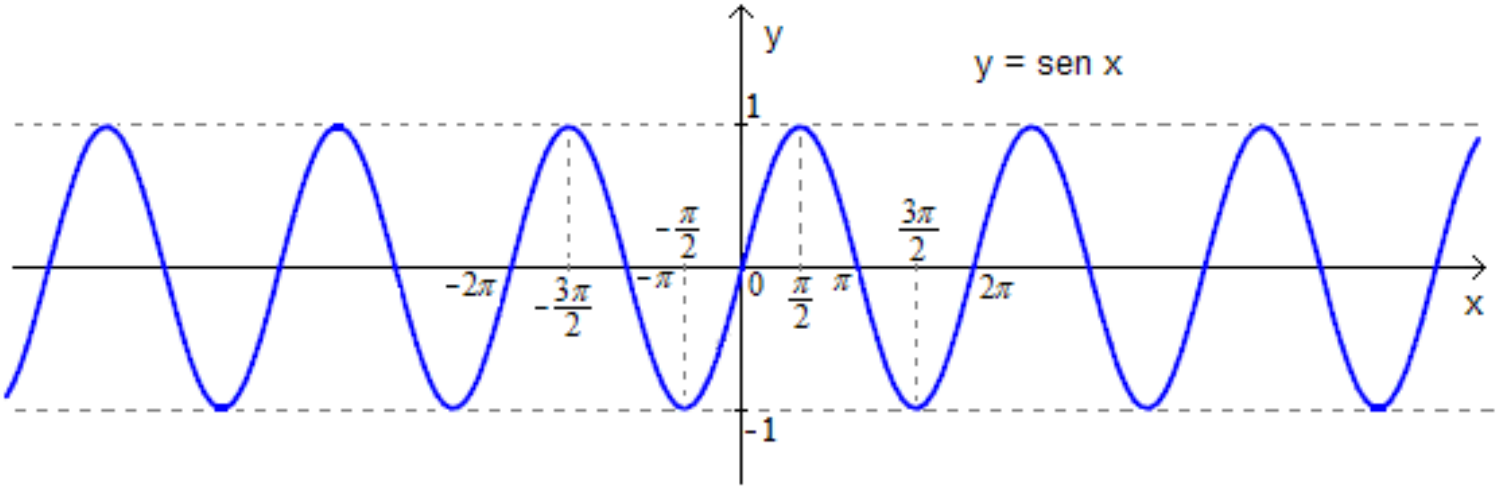


domínio $D = \{x \in \mathbb{R} : x \neq k\pi, k \in \mathbb{Z}\}$
contradomínio $D' =]-\infty, -1] \cup [1, +\infty[$
assímptotas verticais $x = k\pi, k \in \mathbb{Z}$

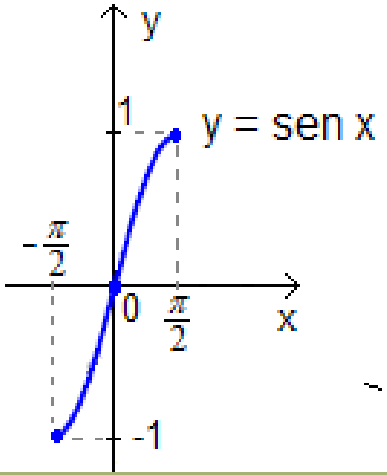
período 2π
função não injectiva
função ímpar
$\text{cosec}(-x) = -\text{cosec}(x)$

Funções Trigonométricas Inversas

Função seno $y = \text{sen}(x)$



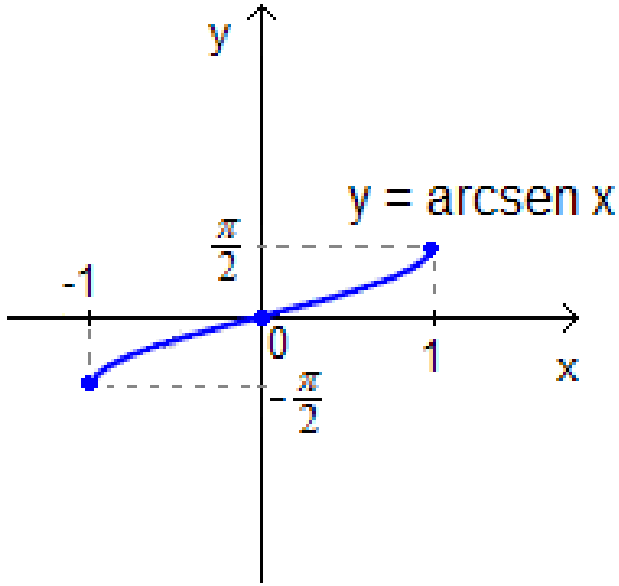
restrição principal $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



Funções Trigonométricas Inversas

Função arco-seno

$$y = \text{arcsen}(x)$$



domínio $D = [-1, 1]$

contradomínio $D' = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

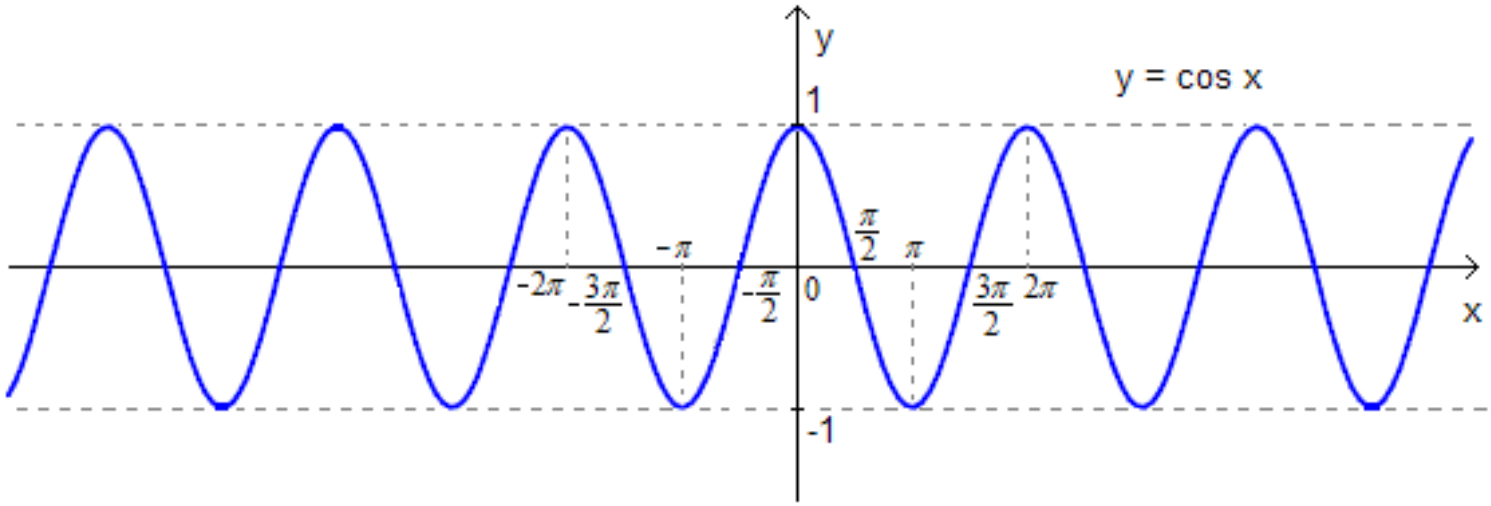
função ímpar $\text{arcsen}(-x) = -\text{arcsen}(x)$

$$\text{sen}(\text{arcsen}(x)) = x \qquad x \in [-1, 1]$$

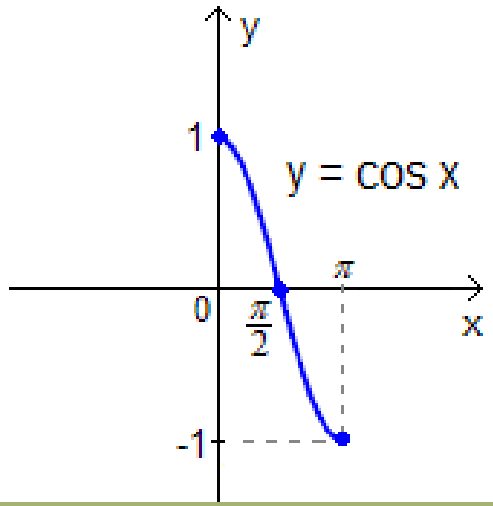
$$\text{arcsen}(\text{sen}(x)) = x \qquad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Funções Trigonométricas Inversas

Função cosseno $y = \cos(x)$



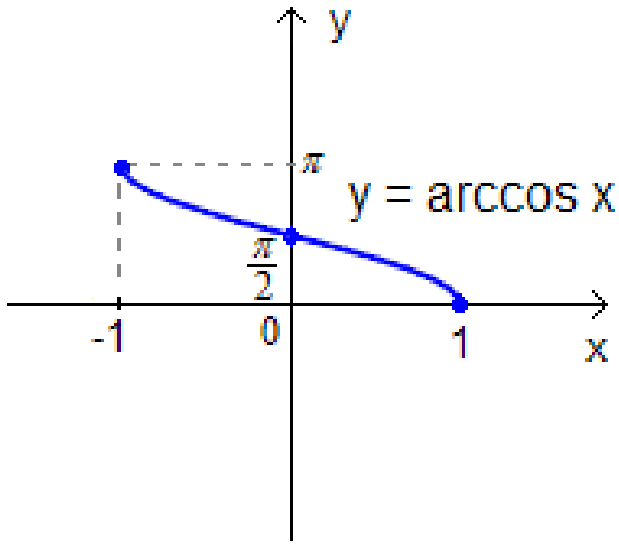
restrição principal $[0, \pi]$



Funções Trigonométricas Inversas

Função arco-cosseno

$$y = \arccos(x)$$



domínio $D = [-1, 1]$
contradomínio $D' = [0, \pi]$

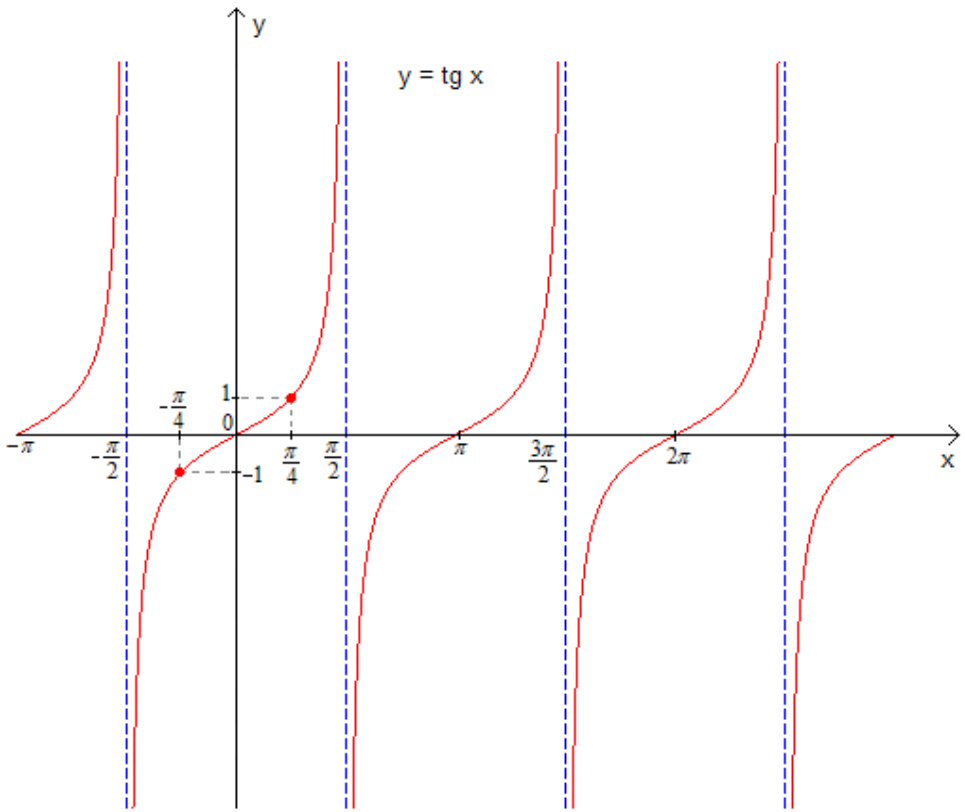
não é função par nem ímpar

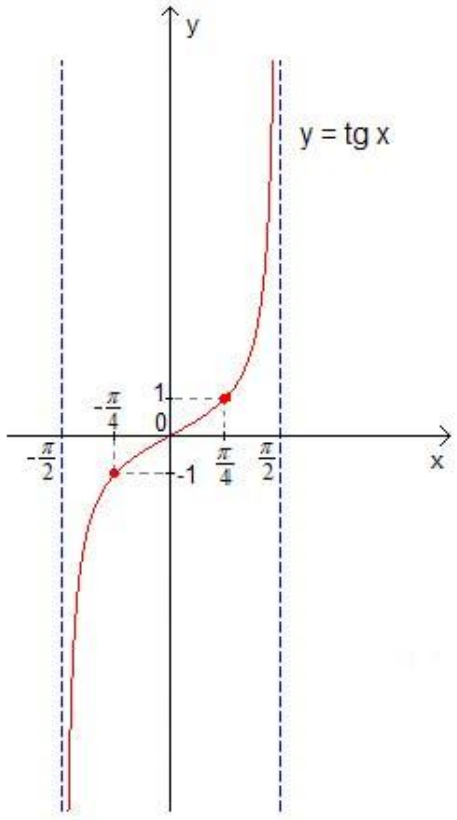
$$\cos(\arccos(x)) = x \qquad x \in [-1, 1]$$
$$\arccos(\cos(x)) = x \qquad x \in [0, \pi]$$

Função tangente

$$y = \text{tg}(x)$$

restrição principal $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

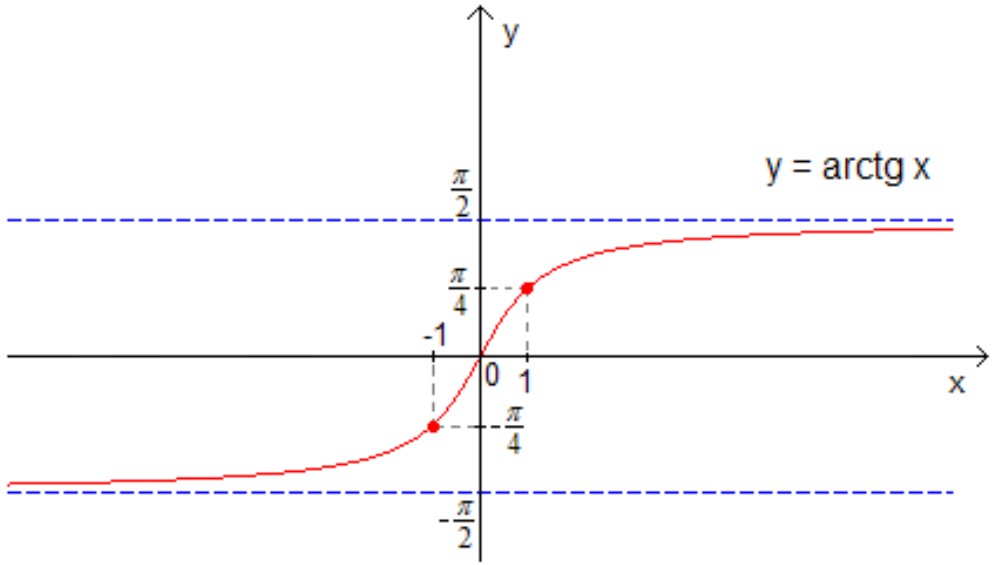




Funções Trigonométricas Inversas

Função arco-tangente

$$y = \text{arctg}(x)$$



domínio $D = \mathbb{R}$
contradomínio $D' = \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

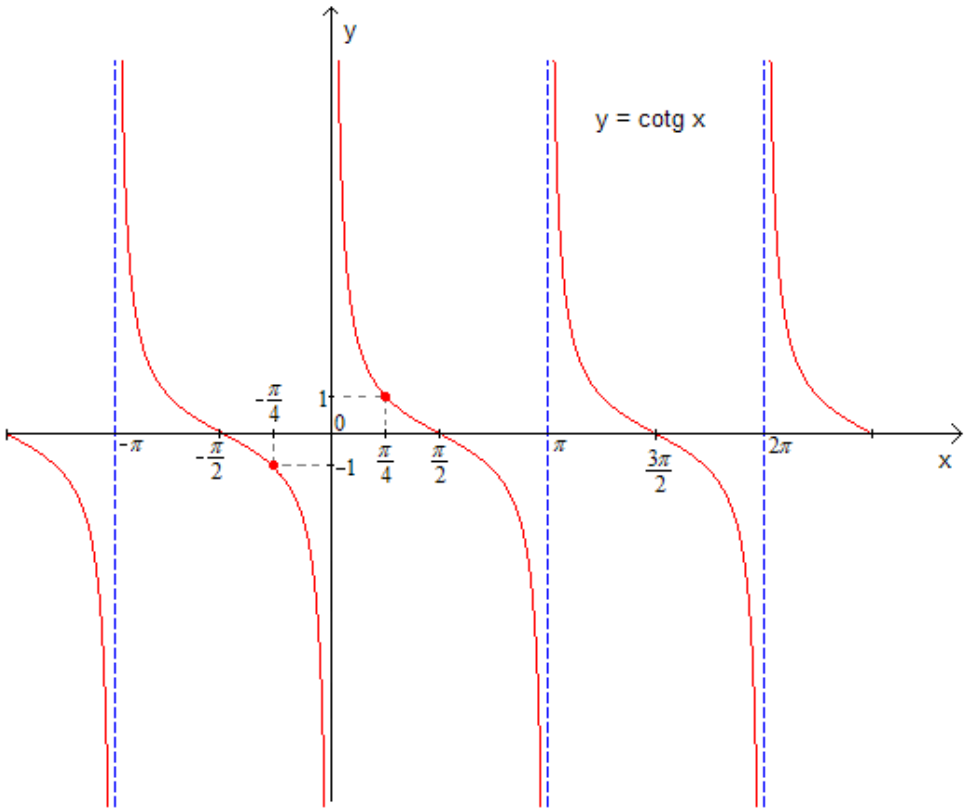
função ímpar $\text{arctg}(-x) = -\text{arctg}(x)$

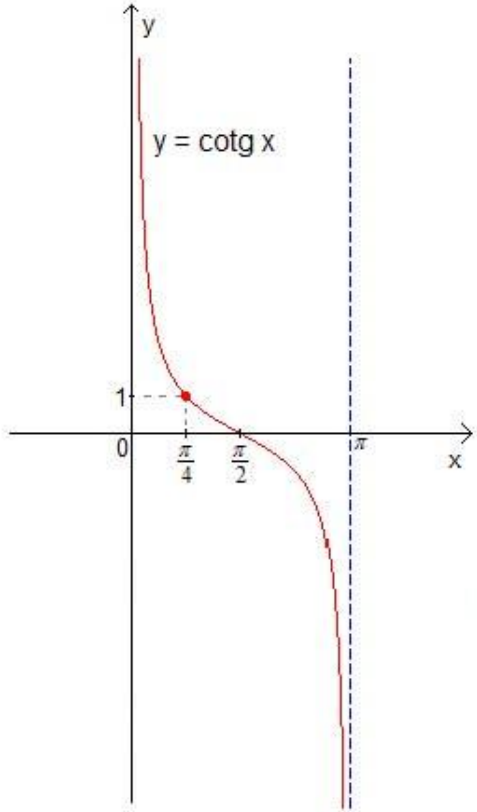
$$\text{tg}(\text{arctg}(x)) = x \qquad x \in \mathbb{R}$$
$$\text{arctg}(\text{tg}(x)) = x \qquad x \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$$

Função cotangente

restrição principal $]0, \pi[$

$$y = \text{cotg}(x)$$

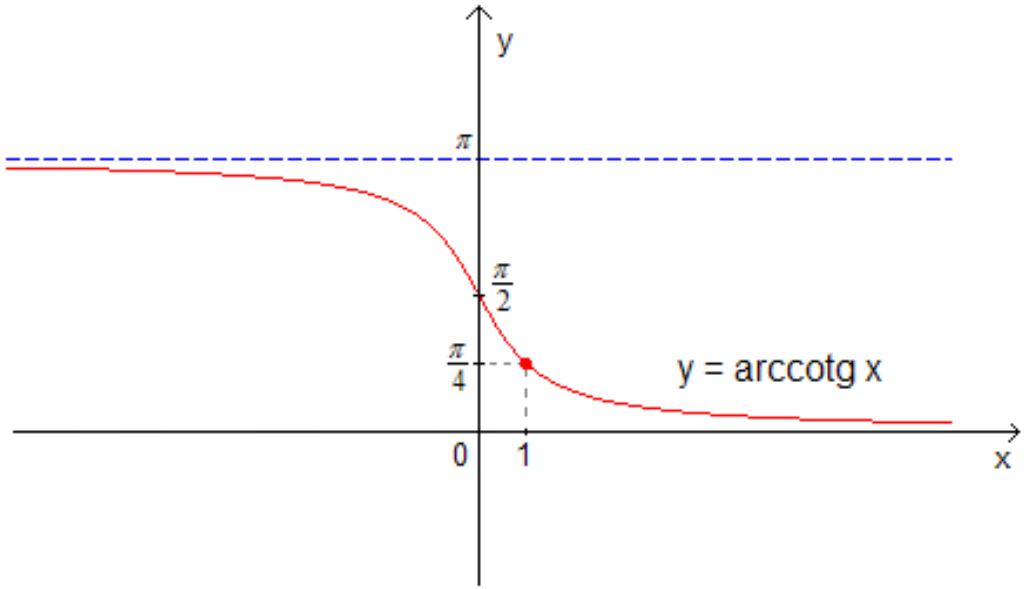




Funções Trigonométricas Inversas

Função arco-cotangente

$$y = \text{arccotg}(x)$$



domínio $D = \mathbb{R}$
contradomínio $D' =]0, \pi[$

não é função par nem ímpar

$$\text{cotg}(\text{arccotg}(x)) = x \qquad x \in \mathbb{R}$$
$$\text{arccotg}(\text{cotg}(x)) = x \qquad x \in]0, \pi[$$